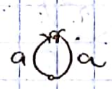
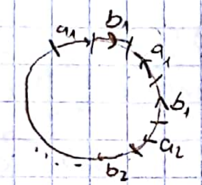
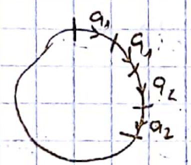
Lecture 3: (from next sunday: Shreiber 007) only on sundays. 11/3/2017

Thm.:

Classification of connected closed surfaces (Dehn-Heegard 1907):

Every closed surface is one of the following.

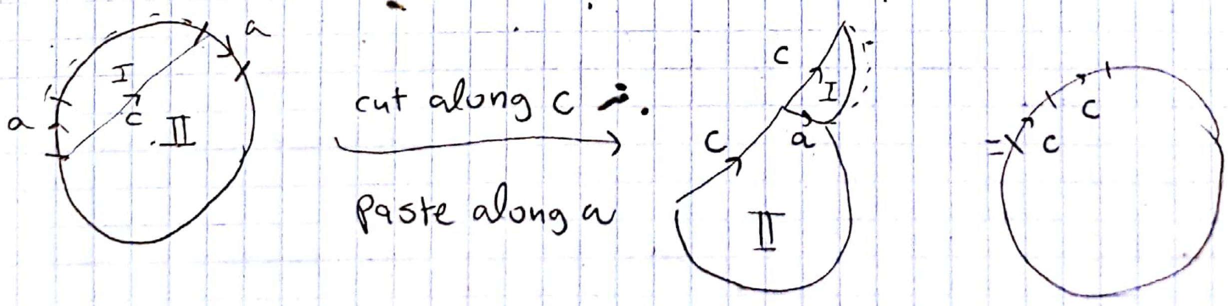
	Canonical form	E.C	orientability	π_1
1) Sphere S^2		2	+	$\{e\}$
2) $\underbrace{T\#T\# \ \#T}_{n}$		$2-2n$	+	$\langle a_1, b_1, \ldots, a_n b_n \mid [a_1,b_1] \ldots [a_n,b_n] \rangle$
3) $\underbrace{P\# \ \ \#P}_{n}$		$2-n$	−	$\langle a_1, \ldots, a_n \mid a_1^2 a_2^2 \ldots a_n^2 \rangle$

Why every connected closed surface is homeomorphic to one of these?

Step 1: Reduction to a single equivalence class of vertices.

Step 2: Crosscap normalization:

All pairs of like oriented edges replaced by adjacent pairs.



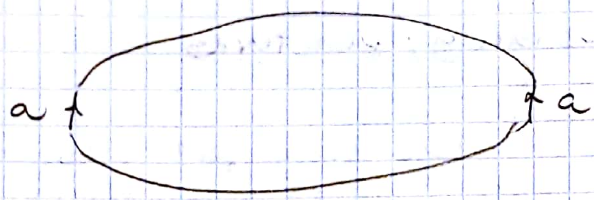
Step 3: Handle normalization.

Claim:

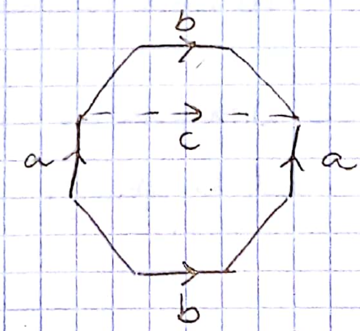
Every pair of opposite oriented edges must occur as "crossed" pairs.

$\ldots a \ldots b \ldots a^{-1} \ldots b^{-1} \ldots$

Pf.:

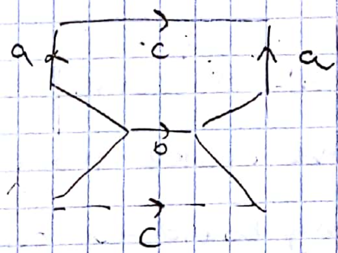


Otherwise, the upper vertices are not glued with the bottom ones.



cut along c

paste along b



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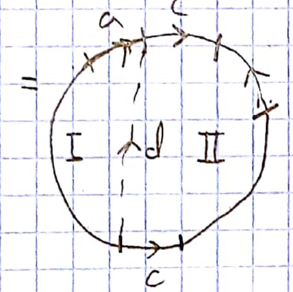
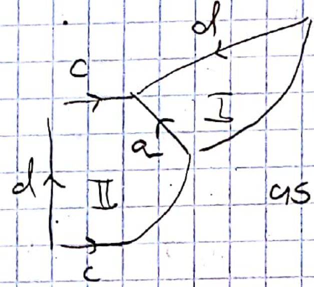


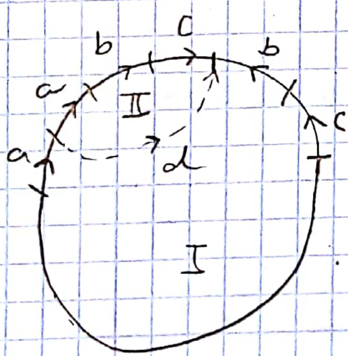
cut along d

paste along a



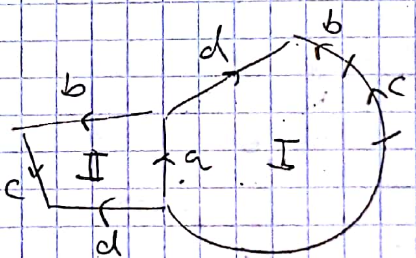
here $c^{-1}dcd^{-1}$ as a subword.

<u>Step 4:</u> <u>If both crosscaps & handles $\Longrightarrow$ only crosscaps.</u>



cut along d

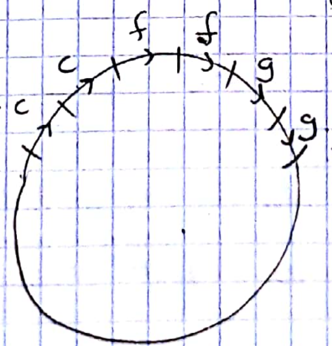
paste along a



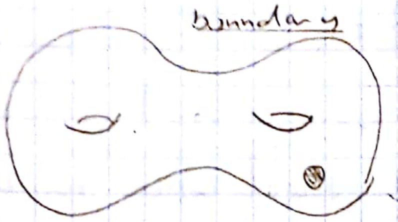
(exercise)

crosscap normalization

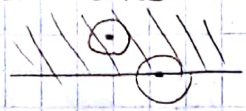
b then c then d



Surfaces with boundary / punctures



where every point looks localy, either like $\mathbb{R}_2$ or like :

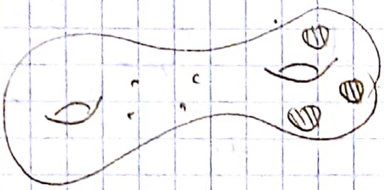


Punctures: points removed from the surface. $\Rightarrow$ no longer compact

classification of connected compact surfaces with punctures:

On top of the choice from the above list, we need to specify

1) how many boundary components $\in \mathbb{N}$
2) how many punctures $\in \mathbb{N}$



Note:

All compact surfaces with a boundary can be embedded into $\mathbb{R}^3$.